Note 1

$x^2 - (\text{二根之和})x + (\text{二根之积}) = 0$ / $x^2 - (sum\ of\ roots)x + (\ product\ of\ roots) = 0$

二根之和 / sum of roots (S.O.R) / $\alpha + \beta$/ $\frac{-b}{a}$ 二根之积 /product of roots (P.O.R) / $\alpha\beta$ / $\frac{c}{a}$

Exercise 1 以不同方式解下列方程式 Solve the following equation by using different methods

$2x^2 - 5x + 2 = 0$

Note	①	②	③	④
	使用计算机 Using Calculator	因式分解 Factorization	公式法 Formula	配方法 Completing the square

Exercise 2 判断下列各方程式的根的性质 Determine the type of roots

两个相异的实根 Two different roots	两个相等的实根 Two equal roots	没有实根 No roots
$2x^2 - 5x + 2 = 0$	$9x^2 + 30x + 25 = 0$	$3x^2 + 4x + 2 = 0$
⚠ … 取 k 的范围值 …	⚠ … 取 k 的值 …	⚠ … 取 k 的范围值 …

关键词 **Key words**

A. 完全平方式 Perfect square expressions
B. 等根 Equal roots
C. 二重根 Repeated roots
D. 有实根 Real roots
E. 切线 Tangent
F. y 值>0 , y 值 <0
G. 与 *x* 轴不相遇 Does not meet the x axes

一元二次方程式的根与系数的关系

Exercise 3

若方程式 $5x^2 + 17x - 12 = 0$ 的一根为 $\frac{3}{5}$， 求其另外一根。

If a root of quadratic equation $5x^2 + 17x - 12 = 0$ is $\frac{3}{5}$, find the other root. [Ans: -4]

Tutorial 1

1. 如果方程式 $5x^2 + 17x - 12 = 0$ 的一根为 $\frac{3}{5}$， 试求其另一根。
   If $\frac{3}{5}$ is a root of quadratic equation $5x^2 + 17x - 12 = 0$, find the value of the other root. [Ans: -4]

2. 如果方程式 $15x^2 + x + m = 0$ 的一根为 $\frac{1}{3}$ ， 试求另一根及 m 之值。
   If $\frac{1}{3}$ is one root of quadratic equation $15x^2 + x + m = 0$, find the value of the other root and value of m.
   [Ans: $\frac{-2}{5}$, $m = -2$]

3. 如果方程式 $8x^2 + x + p = 0$ 的一根为 $\frac{1}{4}$， 试求其另一根及 p 之值。
   If $\frac{1}{4}$ is one root of quadratic equation $8x^2 + x + p = 0$, find the value of the other roots and value of p.
   [Ans: $\frac{-3}{8}$, $p = -1$]

4. 已知 $x^2 - 6x + m = 0$ 的一根比另一根大 2， 试求 m 的值。
   Given one root of quadratic equation $x^2 - 6x + m = 0$ is exceed 2 than the other root, find the value of m .
   [Ans: m = 8]

5. 如果方程式 $4x^2 - px + 2 = 0$ 的一根是另一根的两倍， 试求这两个根及 p 的值。
   One root of quadratic equation $4x^2 - px + 2 = 0$ is twice of the other root, find the value of both of the roots and the value of p.
   [Ans: $\frac{1}{2}$, 1 , $p = 6$]

6. 如果方程式 $x^2 - 7x + 2 = 0$ 的解集为 $\{\alpha, \beta\}$, 试求
The solution of quadratic equation $x^2 - 7x + 2 = 0$ is $\{\alpha, \beta\}$, find
a) $\alpha^2 + \beta^2$ [Ans: 45] b) $\frac{2-\alpha}{3+\beta} + \frac{2-\beta}{3+\alpha}$ [Ans: $\frac{-5}{4}$]

7. 如果 α, β 是方程式 $3x^2 + 5x - 2 = 0$的二根， 试求
If α, β are roots of quadratic equation $3x^2 + 5x - 2 = 0$, find
a) $(\alpha - \beta)^2$ [Ans: $5\frac{4}{9}$] b) $\frac{1}{\alpha} + \frac{1}{\beta}$ [Ans: $\frac{5}{2}$] c) $(\alpha + 1)(\beta + 1)$ [Ans: $\frac{-4}{3}$]

8. 如果 α, β 是方程式 $4x^2 - (5a + 1)x + 5a = 0$ 的两个根， 且 $\beta = 1 + \alpha$， 试求 α, β, a 的可能值。
If α, β are roots of quadratic equation $4x^2 - (5a + 1)x + 5a = 0$, where $\beta = 1 + \alpha$, find the possible values of α, β, a. [Ans: $\alpha = \frac{3}{2}, \beta = \frac{5}{2}, a = 3$ or $\alpha = -\frac{1}{2}, \beta = \frac{1}{2}, a = -\frac{1}{5}$]

Exercise 4 绘出以下图形 **Sketch the graph of the following quadratic functions**

①	②	③
$y = 2x^2 + 5x - 3$	$y = 9x^2 + 12x + 4$	$y = -3x^2 + 4x - 6$

Note 2 一元二次方程式的根的判别式 Determine the type of Roots

	$+x^2$	$-x^2$	b^2-4ac Discriminant
两个相异的实根 、 异根 Two Different roots			
两个相等的实根 Two equal roots			
无实根 No roots			
有实根 Have roots			

Tutorial 2 判别下列各方程式的根的性质 :
Determine the type of root of the following quadratic equation.

1. $4x^2 - 12x + 9 = 0$ [Ans: 等根 equal roots]
2. $5x^2 - 9x + 2 = 0$ [Ans: 异根 different roots]
3. $6x^2 + 7x + 4 = 0$ [Ans:无根 no root]
4. $x^2 + x(x-1) = x - 3$ [Ans:无根 no roots]
5. $x^2 + 20 = 8x$ [Ans: 无根 no root]

Tutorial 3 如果下列方程式都有等根， 试求 *m* 的值。
If roots of each of the quadratic equation are equal roots, find the value of *m*.

1. $mx^2 + 2mx + m = 2x^2$ [Ans: $m = 0$]
2. $x^2 + 2(m+2)x + 9m = 0$ [Ans: $m = 1, 4$]
3. $(m+4)x^2 = (m+3)x + 1$ [Ans: $m = -5$]
4. $4x^2 + 4(m-1)x = -m^2$ [Ans: $m = \frac{1}{2}$]
5. $(m+2)x^2 + 4mx + (2m+3) = 0$ [Ans: $m = 1, \frac{2}{7}$]
6. $x^2 - (m+4)x + 2m + 5 = 0$ [Ans: $m = \pm 2$]
7. $3x^2 + (m+5)x = 2m + 10$ [Ans: $m = -29, -5$]
8. $(m+1)x^2 + 4mx + (2m+3) = 0$ [Ans: $m = -\frac{1}{2}, 3$]
9. $(3x+m)^2 = 12x$ [Ans: $m = 1$]
10. $x^2 - 15 - m(2x-8) = 0$ [Ans: $m = 3, 5$]

Tutorial 4

1. 一元二次方程式 $2x^2 - 1 = 5x + p$ 有异根。 求 p 的范围值。
   The quadratic equation $2x^2 - 1 = 5x + p$ has two different roots. Determine the range of values of p.
   [Ans: $p > \frac{-8}{3}$]

2. 求 h 的值若一元二次方程式 $2x^2 + 12x = 2h - 9$ 有等根。
   Find the value of h if the quadratic equation $2x^2 + 12x = 2h - 9$ has two equal roots. [$h = \frac{-9}{2}$]

3. 证明一元二次方程式$(m+2)x^2 - 5x + 5 = 0$ 无实根。
   Show that the equation $(m+2)x^2 - 5x + 5 = 0$ has no roots if $m > -\frac{3}{4}$.

Note 3

1. $\alpha^2 + \beta^2 =$

2. $\alpha^2 - \beta^2 =$

3. $(\alpha - \beta)^2 =$

4. $\alpha - \beta =$

5. $\alpha^3 + \beta^3 =$

6. $\alpha^3 - \beta^3 =$

7. $\alpha^4 + \beta^4 =$

8. $\alpha^4 - \beta^4 =$

9. $\alpha^6 + \beta^6 =$

10. $\alpha^8 + \beta^8 =$

Tutorial 5

设α 与 β 是一元二次方程式$x^2-7x+2=0$ 的根，$\alpha>\beta$。 求以下各别的值。
If α and β are two roots of the quadratic equation $x^2-7x+2=0$, $\alpha>\beta$. Find the value of the following:

1. $(\alpha+1)(\beta+1)$ [Ans:10]
2. $\frac{1}{\alpha}+\frac{1}{\beta}$ [Ans: $\frac{7}{2}$]
3. $(\alpha-\beta)^2$ [Ans: 41]
4. $\alpha-\beta$ [Ans: $-\sqrt{41}$]
5. $\alpha^2+\beta^2$ [Ans: 45]
6. $\frac{2-\alpha}{\beta}+\frac{2-\beta}{\alpha}$ [Ans: $-\frac{31}{2}$]

Tutorial 6

设α 与 β 是一元二次方程式$2x^2-4x-3=0$ 的根，$\alpha>\beta$。 求以下各别的值。
If α and β are two roots of the quadratic equation $2x^2-4x-3=0$, $\alpha>\beta$. Find the value of the following:

1. $\alpha^2+\beta^2$ [Ans: 7]
2. $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ [Ans: $-\frac{14}{3}$]
3. $(\alpha-\beta)^2$ [Ans: 10]
4. $\alpha-\beta$ [Ans: $\sqrt{10}$]
5. $\alpha^2-\beta^2$ [Ans: $2\sqrt{10}$]
6. $\alpha^4-\beta^4$ [Ans: $14\sqrt{10}$]

Tutorial 7

若α 与 β 是一元二次方程式$2x^2 - 5x - 7 = 0$ 的根。根据以下新的根以创建一个新的一元二次方程式。
If α and β are two roots of the quadratic equation $2x^2 - 5x - 7 = 0$. Form a quadratic equation with the following roots :

1. α^2 , β^2 [Ans: $4x^2 - 53x + 49 = 0$]

2. $\frac{\alpha}{\beta} , \frac{\beta}{\alpha}$ [Ans: $14x^2 + 53x + 14 = 0$]

3. $\alpha + \frac{1}{\beta} , \beta + \frac{1}{\alpha}$ [Ans: $14x^2 - 25x - 25 = 0$]

Tutorial 8

若α 与 β 是一元二次方程式$2x^2 - x - 7 = 0$ 的根。根据以下新的根以创建一个新的一元二次方程式。
If α and β are two roots of the quadratic equation $2x^2 - x - 7 = 0$. Form a quadratic equation with the following roots:

1. $\alpha + 3$, $\beta + 3$ [Ans: $2x^2 - 13x + 14 = 0$]
2. $\frac{\alpha}{\beta} , \frac{\beta}{\alpha}$ [Ans: $14x^2 + 29x + 14 = 0$]
3. $2\alpha - \beta , 2\beta - \alpha$ [Ans: $2x^2 - x - 92 = 0$]
4. $\alpha + 2\beta , \beta + 2\alpha$ [Ans: $2x^2 - 3x - 6 = 0$]
5. $\alpha^2 + \beta^2 , 2\alpha\beta$ [Ans: $4x^2 - x - 203 = 0$]
6. $\alpha + \frac{1}{\alpha} , \beta + \frac{1}{\beta}$ [Ans: $14x^2 - 5x - 82 = 0$]

Tutorial 9

以α 和 β 表示以下各一元二次方程的二根之和与二根之积 。 [答案请观看视频]
Express sum of roots and product of roots in terms of α , β of each of the following quadratic equation.

答案只允许 $\alpha+\beta$ ， $\alpha\beta$ 及数字而已。 $\alpha+\beta$ ， $\alpha\beta$ and numbers are allowed in answer.

1. $\alpha^2\beta$ ， $\beta^2\alpha$

2. $\left(\frac{\alpha}{\beta}\right)^2$ ， $\left(\frac{\beta}{\alpha}\right)^2$

3. $\sqrt{\frac{\alpha}{\beta}}$ ， $\sqrt{\frac{\beta}{\alpha}}$

4. $\frac{\alpha}{\beta-1}$ ， $\frac{\beta}{\alpha-1}$

5. $\frac{1+\alpha}{1-\alpha}$ ， $\frac{1+\beta}{1-\beta}$

6. $(\alpha+1)$ ， $(\beta+1)$

7. $\left(\alpha+\frac{1}{\beta}\right)$ ， $\left(\beta+\frac{1}{\alpha}\right)$

8. $\frac{\alpha^3}{\beta}$ ， $\frac{\beta^3}{\alpha}$

9. $(\alpha-1)$ ， $(\beta-1)$

10. α^2 ， β^2

11. $\frac{1}{\alpha}$ ， $\frac{1}{\beta}$

12. $1-\frac{1}{\alpha}$ ， $1-\frac{1}{\beta}$

Tutorial 10

1. 以一般式写出一元二次方程式$\frac{5}{x-2}=2x-1$。
   Write the quadratic equation $\frac{5}{x-2}=2x-1$ in the general form. [Ans: $2x^2-5x-3=0$]

2. 若 3 是一元二次方程式$2x^2-7x+k=0$的根，求 k 的值。
   If 3 is one of the roots of the equation $2x^2-7x+k=0$, find the value of k. [Ans: $k=3$]

3. 若$\frac{1}{2}$是一元二次方程式$kx^2-2kx+k-1=0$的根，求 k 的值。
   If $\frac{1}{2}$ is one of the roots of the quadratic equation $kx^2-2kx+k-1=0$, find the value of k .
   [Ans: $k=4$]

4. 若$\frac{1}{m}$是一元二次方程式$mx^2+7x-2m=0$ 的根，求m 的值。
   If $\frac{1}{m}$ is one of the roots of the quadratic equation $mx^2+7x-2m=0$, find the values of m .
   [Ans: $m=\pm 2$]

5. 已知 2 是一元二次方程式$x^2+3x-p=0$ 的根，p 是常数， 求另外一根的值。
   Given that 2 is one of the roots of the quadratic equation $x^2+3x-p=0$, where p is a constant, find the value of the other root. [Ans: -5]

6. 解一元二次方程式$(4x+2)^2-3x(4x+2)-18=0$.
Solve the quadratic equation $(4x+2)^2-3x(4x+2)-18=0$. [Ans: $1\ or\ -3\frac{1}{2}$]

7. 以配方法解一元二次方程式$k^2-k-12=0$.
Solve the quadratic equation $k^2-k-12=0$ by completing the square. [Ans: -3 or 4]

8. 求一元二次方程式$2x^2-9x+6=0$的根，答案以二位小数写出。
Find the roots of the quadratic equation $2x^2-9x+6=0$. State your answer correct to two decimal places. [Ans: 3.69 or 0.81]

9. 已知$f(x)=2x+\frac{3}{x}$,$x\neq 0$， 求 k 的值当$f\left(\frac{1}{k}\right)=4k-1$。
Given that $f(x)=2x+\frac{3}{x}$,$x\neq 0$, find the values of k such that $f\left(\frac{1}{k}\right)=4k-1$.
[Ans: $k=-1\ or\ 2$]

Tutorial 11

1. 一元二次方程式的二根之和是$\frac{-2}{5}$， 二根之积是$\frac{-3}{5}$。求次一元二次方程式。
   The sum of the roots of a quadratic equation is $\frac{-2}{5}$ while the product of the roots of the equation is $\frac{-3}{5}$. State the quadratic equation. [Ans: $5x^2 + 2x - 3 = 0$]

2. 求一元二次方程式当根为$\frac{-2}{3}$ 及 $\frac{3}{2}$。
   Form a quadratic equation whose roots are $\frac{-2}{3}$ and $\frac{3}{2}$. [Ans: $6x^2 - 5x - 6 = 0$]

3. 求一元二次方程式当根为二重根 4。
   Form a quadratic equation that has a repeated root of 4. [Ans: $x^2 - 8x + 16 = 0$]

4. 已知一元二次方程式$3x - (x + 2) = 3x(x + 2)$， 求
   Given the quadratic equation $3x - (x + 2) = 3x(x + 2)$, find
   a. 二根之和 The sum of roots [Ans: $\frac{-4}{3}$]

   b. 二根之积 The product of roots [Ans: $\frac{2}{3}$]

5. 一元二次方程式$px^2 - px + 2 = 0$的一根是另外一根的两倍， 求 p 的值。
One of the roots of the quadratic equation $px^2 - px + 2 = 0$ is twice the other root. Find the value of p.
[Ans: $p = 9$]

6. 一元二次方程式$px^2 - px + 2 = 0$的一根是另外一根的两倍， 求 a, b 与 c 的关系。
If one of the roots of the quadratic equation $ax^2 + bx + c = 0$ is two times the other root, find the expression that relates a, b and c. [Ans: $2b^2 = 9ac$]

7. 若 m 及 n 是一元二次方程式$x^2 + 2x - 8 = 0$的根， 求以 2m 及 2n 为根的一元二次方程式。
If m and n are the roots of the quadratic equation $x^2 + 2x - 8 = 0$, form a quadratic equation that has the roots 2m and 2n. [Ans: $x^2 + 4x - 32 = 0$]

8. 一元二次方程式$x^2 - px + 8 = 0$的一根是另一根的平方。求 p 的值。
One root of the quadratic equation $x^2 - px + 8 = 0$ is the square of the other root. Find the value of p .
[Ans: $p = 6$]